

CURRICULUM

Paolo Piccinni

A. BASICS

Personal data: Born in Roma, 1952.
Married in 1989 with Simonetta Schioppa. Two children: Marco, 1990 and Laura, 1993.
1976: Laurea in Matematica, Università di Roma, thesis advisor Beniamino Segre.
1977 - 1981: CNR fellowship at Sapienza Università di Roma, research director Enzo Martinelli.
1981 - 1987: Researcher, Sapienza Università di Roma.
1987 - 1990: Associate Professor, Università di Salerno.
1990 - 2008: Associate Professor, Sapienza Università di Roma.
2008 - : Full Professor, Sapienza Università di Roma.

B. RESEARCH ACTIVITY

- Research periods and visiting positions include: Univ of Illinois at Urbana-Champaign (1982-83), Michigan State Univ (1984), E. Schroedinger Institute Vienna (1994 and 1999), Univ of Oxford (1995), MPI-Bonn (2001 and 2003), IHES (2002), Univ of New Mexico (2002), Centro De Giorgi Pisa (2004).
- Talks at international meetings and at visited universities include: "12th MiniMeeting on Differential Geometry", Guanajuato, Mexico 2020; "Max LX - Miniworkshop on non Kählerian Geometry", Florence 2019; "New Perspectives in Differential Geometry: Special metrics and Quaternionic Geometry", INdAM Roma 2015; "Real and Complex Differential Geometry", Bucharest 2013; "Geometric Structures on Complex Manifolds", Steklov Institute Moscow 2011; "Journée de Géométrie à l'honneur de P. Gauduchon" Florence 2005; "Differential Geometry and Topology" Centro E. De Giorgi, Pisa 2004; University of New Mexico 2002; First joint Meeting AMS-UMI, Pisa 2002; International Congress on Differential Geometry in memory of A. Gray, Bilbao 2000; "Holonomy Groups in Differential Geometry" E. Schrödinger Int. Inst., Vienna 1999; "Complex Methods in Differential Geometry" ICMS Edinburgh 1997; "Conference on Differential Geometry", Budapest 1996; "Quaternionic and Hyperkähler Manifolds" E. Schrödinger Int. Inst., Vienna 1994; "Quaternionic structures in Mathematics and Physics", Trieste SISSA 1994; Univ of Debrecen 1987; Luminy "Geometry and Topology of Submanifolds" 1987; Michigan State Univ 1984; Univ of Illinois at Urbana-Champaign 1982; Oberwolfach "Differentialgeometrie im Grossen" 1981.
- Organized meetings include: *Workshop INdAM Kähler and Sasakian Geometry*, Roma 2009, dedicated to the memory of Krzysztof Galicki; *Second Meeting on Quaternionic Structures in Mathematics and Physics*, Roma 1999.

C. SELECTED PUBLICATIONS

- [1] *Clifford systems, Clifford structures, and their canonical differential forms* (with K. Boydon), Abh. Math. Sem. Univ. Hamburg, 91 (2020), 15 pp. doi.org/10.1007/s12188-020-00229-5
 - [2] *An even Clifford diamond* (with G. Arizmendi and R. Herrera), Ann. Gl. An. Geom., 57 (3), (2020), 465-487.
 - [3] *Parallelizations on products of spheres and octonionic geometry* (with M. Parton), Complex Manifolds, 6 (2019), 138-149.
 - [4] *The rôle of Spin(9) in octonionic geometry* (with M. Parton), Special Issue of Axioms "Applications of Differential Geometry", A. Fino ed., 7 72 (2018), 32 pp.
 - [5] *On some Grassmannians carrying an even Clifford structures*, Diff. Geom. and Appl., 59 (2018), 122-137.
 - [6] *On the cohomology of some exceptional symmetric spaces*, in Special Metrics and Groups Actions in Geometry, Springer-INdAM Series, 23 (2017), chapter 12.
 - [7] *Clifford systems in octonionic geometry*, with M. Parton and V. Vuletescu, Rend. Seminario Matematico di Torino, volume in memory of Sergio Console, 74 (2016), 267-288.
 - [8] *The even Clifford structure of the fourth Severi variety*, with M. Parton, Complex Manifolds, 2 (2015), 89-104.
 - [9] *Spin(9) geometry of the octonionic Hopf fibration*, with L. Ornea, M. Parton and V. Vuletescu, Transformation Groups, 18 (2013), 845-864.
 - [10] *Spheres with more than 7 vector fields: all the fault of Spin(9)*, with M. Parton, Linear Algebra and Appl., 438 (2013), 1113-1131.
 - [11] *Spin(9) and almost complex structures on 16-dimensional manifolds*, with M. Parton, Ann. Gl. An. Geom. 41 (2012), 321-345.
 - [12] *Self-dual Einstein orbifolds with few symmetries as quaternion Kaehler quotients*, with L. Bisconti, J. Geom. Physics, 60 (2010), 8 - 22.
 - [13] *Locally conformal parallel G_2 and Spin(7) manifolds*, with S. Ivanov and M. Parton, Math. Res. Letters, **13** (2006), 1001-1011.
 - [14] *Reduction of Vaisman structures in complex and quaternionic geometry*, with R. Gini, L. Ornea and M. Parton, J. Geom. Phys., **56** (2006), 2501-2522.
 - [15] *Toric self dual Einstein metrics as quotients*, with C. P. Boyer, D. Calderbank and K. Galicki, Comm. in Math. Physics, **253** (2005), 337-370.
 - [16] *3-Sasakian Geometry, Nilpotent Orbits and Exceptional Quotients*, with C. P. Boyer and K. Galicki, Ann. Gl. An. Geom., **21** (2002), 85-110.
 - [17] *Cayley 4-frames and a quaternion Kähler reduction related to Spin(7)*, with L. Ornea, in "Global Differential Geometry: The Mathematical Legacy of Alfred Gray" Contemp. Math. 288, AMS (2001).
 - [18] *Foliations with transversal quaternionic structure*, with I. Vaisman, Ann. di Mat. Pura e Appl., **180** (2001), 303-330.
 - [19] *On some moment maps and induced Hopf bundles in the quaternionic projective space*, with L. Ornea, Int. J. Math. **11** (2000), 925-942.
 - [20] *The Geometry of positive locally quaternion Kähler manifolds*, Ann. Gl. An. Geom., **16** (1998), 255-272.
 - [21] *Locally conformal Kähler structures in quaternionic geometry*, with L. Ornea, Trans. Am. Math. Soc., **349** (1997), 641-655.
 - [22] *Induced Hopf bundles and Einstein metrics*, with L. Ornea, New Developments in Differential Geometry, Budapest 1996, Kluwer Publ. (1998), 295-305.
 - [23] *On the infinitesimal automorphisms of quaternionic structures*, J. Math. Pures Appl., **72** (1993), 593-605.
 - [24] *On some classes of 2-dimensional Hermitian manifolds*, J. Math. Pures Appl., **69** (1990), 227-237.
 - [25] *Finite type submanifolds and finite type Gauss maps*, with B.Y. Chen, Proc. Luminy Meeting "Geometry and Topology of Submanifolds", World Sc., (1989), 29-37.
 - [26] *Submanifolds with finite type Gauss map*, with B.Y. Chen, Bull. Austr. Math. Soc., **35** (1987), 161-186.
 - [27] *The canonical foliations of a locally conformal Kähler manifold*, with B.Y. Chen, Ann. di Mat. Pura e Appl., **141** (1985), 283-305.
 - [28] *A Weitzenböck formula for the second fundamental form of a Riemannian foliation*, Rend. Acc. Naz. Lincei, **77** (1984), 102-110.
 - [29] *A generalization of symplectic Pontrjagin classes to vector bundles with structure group $Sp(n)Sp(1)$* , with G. Romani, Ann. di Mat. Pura e Appl., **133** (1983), 1-18.
 - [30] *Dieudonné determinant and invariant real polynomials on $gl(n,H)$* , Rend. di Mat., **2** (1982), 31-45.
 - [31] *Quaternionic differential forms and symplectic Pontrjagin classes*, Ann. di Mat. Pura e Appl., **129** (1981), 57-68.
- Review of the book *The Many Facets of Geometry - A tribute to Nigel Hitchin*, edited by Oscar Garcia Prada, Jean Pierre Bourguignon and Simon Salamon, Metron Int. J. of Statistics, **69**, (2011), 223-226.
 - Reviews of about 190 items for Mathematical Reviews of AMS, years 1986-2016.

C. STUDENTS

Here is a list of students whose various types of thesis I supervised.

- *Ph D students:* Maurizio Parton (Pisa, 2001), Luca Bisconti (Roma Tor Vergata, 2007), Kai Brynne Boydon (Univ. of the Philippines, 2020).
- *Master students:* Alessandra Mastrandrea, Assunta De Rita, Rosetta Ragno, Gerardo De Luna, Loredana Caso (all at University of Salerno); Daniele Tatti, Maurizio Parton, Rossella De Gaetano, Lucia Illuminati, Fabiana Domizi, Maria Cristina Giorgi, Mariapia Carattoli, Antonella Parrillo, Corrado Ceccarelli, Giulio Alberti, Andrea Morgia, Mario Principato, Alessandra Capotosti, Sonia Mariani (Roma Sapienza).
- *Bachelor students:* Luca Moci, Luigi Lombardo, Matteo Portelli, Silvia Pragliola, Marta Spiti, Arseny Gurin, Laura Eugeni, Adriana Schiani, Maria Chiara Bertini, Susanna Risa, Cristian Pasquinati, Aurelio Carlucci, Ivan D'Annibale, Bruno Federici, Nastassja Cipriani, Francesca Gea Mainini, Francesca Carocci, Paolo Trani, Leonardo Alese, Valeria Bertini, Giulia Grassi, Daniele Albanese, Andrea Peruzzi, Eldar Ceresini, Matteo Bortolotti, Andrea Nicolanti, Paolo Bucci, Alessandro Di Marco, Flavia Scinicarelli, Marta D'Ingillo, Carla Pintore, Francesco Ciccone, Chiara Petrucci, Stefano Santi Laurini, Giacomo Troiano, Pier Paolo Penna (Roma Sapienza).

In 2015 I was awarded for excellent teaching, a recognition granted yearly to about 5% of Faculty of Science, Sapienza Università di Roma.

From 2009 to 2020 I served as academic coordinator of the students' international mobility (including Erasmus mobility) at Department of Mathematics, Sapienza Università di Roma.

D. RECENT RESEARCH INTERESTS

My recent interests have been concerned with the geometry of even Clifford structures of exceptional type, one of the motivations are in my 2006 work [13] on locally conformal parallel G_2 and $Spin(7)$ manifolds. I will now briefly describe some of the themes in the recent papers [1], ... , [11] (most of them with M. Parton, some also with L. Ornea and V. Vuletescu, one with G. Arizmendi and R. Herrera, one with K. Boydon).

In 2001 Th. Friedrich showed that some $Spin(9)$ weakened holonomies on Riemannian manifolds M^{16} can be described in terms of the datum of a vector bundle E over M locally generated by nine self adjoint anti-commuting involutions I_1, \dots, I_9 . The representation $Spin(9) \subset SO(16)$ was indeed in the original M. Berger holonomy list, but in fact only weakened holonomy $Spin(9)$ is allowed on non-symmetric spaces. There is a 1-dimensional invariant component under $Spin(9)$ in the space of 8-forms in \mathbb{R}^{16} , and any of its generators gives rise to a canonical 8-form $\Phi_{Spin(9)}$. Such a $\Phi_{Spin(9)}$ had been constructed (M. Berger, 1972, and independently R. Brown - A. Gray, 1972), through integrals over S^8 and over $Spin(8)$, respectively. In the paper [6] a natural algebraic approach to $\Phi_{Spin(9)}$ is proposed, somehow following what one usually does for defining the canonical 3-form associated with G_2 or the 4-forms associated with a quaternionic structures $Sp(n) \cdot Sp(1)$ and with a $Spin(7)$ structure, basing on the Friedrich $Spin(9)$ vector bundle E . Accordingly, the 702 non-zero monomials in $\Phi_{Spin(9)} \in \Lambda^8(\mathbb{R}^{16})$, are computed starting from the Kähler 2-forms associated with the (local) almost complex structures $J_{\alpha\beta} = I_\alpha \circ I_\beta$.

Some applications of this approach are given in the papers [9] and [10]. Namely, [9] completes a kind of scheme of description for metrics which are locally conformal parallel with respect to the G -structures one consider for Riemannian holonomies. Apart from locally conformal complex Kähler metrics, whose study is definitely still in progress, I studied locally conformal hyperkähler and locally conformal quaternion Kähler metrics in the 1997 Transaction paper [21] with L.Ornea, and locally conformal parallel G_2 and $Spin(7)$ metrics in the MRL paper [13] with S. Ivanov and M. Parton. In [9] contains, a structure theorem for

compact locally conformal parallel Spin(9) manifolds is proved as well as the non-existence on nowhere zero vertical vector fields in the octonionic Hopf fibration $S^{15} \rightarrow S^8$. The paper [10] gives instead an elementary application of the almost complex structures $J_{\alpha\beta}$ to the construction of a maximal orthonormal system of tangent vector fields on sphere of any odd dimension. This is a linear algebra result, obtained through an inductive procedure, starting from dimension 15, and taking into account the complex, quaternionic or octonionic structure that can be allowed on the ambient Euclidean space.

The papers [6], [7], [8] deal with some geometries that extend the Spin(9) one. Namely, [8] focuses on the exceptional Hermitian symmetric space of compact type E III, whose projective algebraic model is the so-called fourth Severi variety in $\mathbb{C}P^{26}$, known in projective algebraic geometry for remarkable properties concerning its secant and tangent lines. Its Riemannian holonomy group is $\text{Spin}(10) \cdot \text{U}(1)$, and the techniques already described allow to construct on it a similar canonical differential 8-form $\Phi_{\text{Spin}(10) \cdot \text{U}(1)}$. This 8-form is shown to be related with some nice algebraic cycles on the manifolds that were pointed out by A. Iliev and L. Manivel in relation with its Chow ring. On the other hand, the geometry of this complex projective manifold fits in one of the exceptional even Clifford structures, this latter a kind of unifying notion introduced by A. Moroianu and U. Semmelmann and presently in evolution in several contexts. Further exceptional even Clifford symmetric spaces are the quaternion Kähler Wolf space E VI, and also the exceptional symmetric space E VIII, among the ones of compact type that has the largest dimension. It is shown in the papers [6] and [7] that on E III, E VI and E VIII the even Clifford structure turns out to be *essential*, meaning that, in contrast with the Spin(9) case, such an even Clifford structure cannot be obtained through a certain number of local self adjoint anticommuting involutions. Nevertheless, a canonical 8-form is defined in all the cases, and one can see how much the cohomology of all these exceptional symmetric spaces turns out to be primitive, i.e. independent on all the canonical differential forms associated with their geometries.

My last papers [1] [2], [3], [4], [5], are mainly devoted to Clifford systems and even Clifford structures on Riemannian manifolds of dimension 8, 16, and multiples of 16. I studied in particular classes of examples and individual examples of manifolds with such structures, like some classes of real, complex and quaternionic Grassmannians, then again some exceptional symmetric spaces of compact type, and finally some parallelizable examples. Topological aspects have been focused. Bundle constructions have been carried out, by showing that both the mentioned Clifford properties give often rise to rich diagrams of Einstein manifolds, extending the so-called diamond diagram of positive quaternion Kähler geometry.

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